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Research Article

Estimation of Soil Electrical Properties in a Multilayer Earth Model with Boundary Element Formulation

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This paper presents an efficient model for estimation of soil electric resistivity with depth and layer thickness in a multilayer earth structure. This model is the improvement of conventional two-layer earth model including Wenner resistivity formulations with boundary conditions. Two-layer soil model shows the limitations in specific soil characterizations of different layers with the interrelationships between soil apparent electrical resistivity (ρ) and several soil physical or chemical properties. In the multilayer soil model, the soil resistivity and electric potential at any points in multilayer anisotropic soil medium are expressed according to the variation of electric field intensity for geotechnical investigations. For most soils with varying layers, multilayer soil resistivity profile is therefore more suitable to get soil type, bulk density of compacted soil and to detect anomalous materials in soil. A boundary element formulation is implemented to show the multilayer soil model with boundary conditions in soil resistivity estimations. Numerical results of soil resistivity ratio and potential differences for different layers are presented to illustrate the application, accuracy, and efficiency of the proposed model. The nobility of the research is obtaining multilayer soil characterizations through soil electric properties in near surface soil profile.

1. Introduction

Soil monitoring using electrical resistivity has been widely applied to many geotechnical and other engineering problems to investigate near surface soil profile [1]. In particular, direct

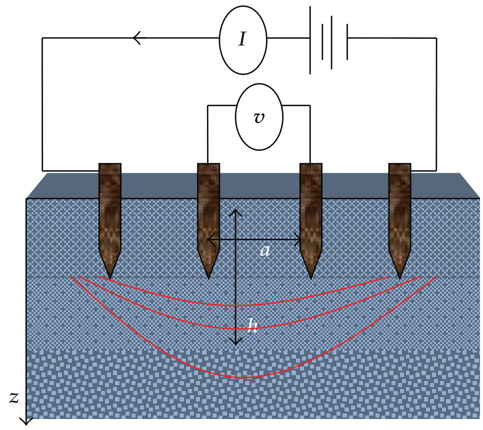

Figure 1: Soil resistivity measurements in near surface profile.

current (DC) resistivity monitoring has been actively used in geotechnical investigations, since the resistivity of subsurface material is easily affected by conductive or resistive fluid injection [2–4].

Near surface soil characterizations and soil strength determinations are prerequisite in highway and road engineering including construction of highway embankments, earth dams, geotechnical engineering, and other divisions of civil engineering. The soil electrical resistivity and electric potential differences can be used for determining the specific soil characteristics [5–7] corresponding to the depth of soil. The model of soil electric properties with multilayer earth structure will be able to perform specified soil characterizations through laver-by-layer earth analysis in geotechnical investigations. This study will encourage innovations in multilayer soil resistivity profile to scrutinize arrangement of specific characteristics of near surface soil such that the objectives of the geotechnical investigation can be realized.

A two-layer soil model is generally used for nonhomogeneous soil characterizations in geoelectric engineering. The soil resistivity measurements are commonly done by four-probe Wenner method. Current is passed through two points current sources, I, at the surface of the earth. Measurements of voltage, V, between two points at the surface of the earth are taken between these points as shown in Figure 1. The electrical resistance, R, is obtained by dividing V by I according to the Ohm's law. The basic principle of the soil resistivity, ρ measurement system is that when a constant voltage is applied to one of the two probes placed in the soil the current that flows between the probes is inversely proportional to the resistance of the soil [8].

Though the apparent resistivity estimation using the conventional Wenner four-electrode measurements is relatively simple and its usefulness has always been recognized, it is not a method explicitly directed at multilayer soil structures. The resulting data still shows lack of accuracy for two-layer soil model. The limitations are found especially in terms of the interrelationships between soil apparent electrical resistivity (ρ) and several soil physical properties. In response to this situation, we have calculated the true resistivity in a horizontally multilayer earth model using the four-probes method.

This study includes resistivity curve features for a variety of earth parameters and has made possible the investigations of techniques for estimating these earth parameters.

Soil electric potentiality and thickness are also considered to obtain efficient outcomes with multilayer analysis. The ability to perform multilayer soil resistivity profile is useful to get bulk density of compacted soil, to estimate the type of soil and rocks, to mark granular soil and groundwater surface, to detect anomalous materials, and to estimate the depths of bedrock surfaces in geotechnical engineering.

In this work, boundary element method (BEM) is implemented on soil electric potential for electric field analyses which is the physical quantity of interest to analysts [9, 10]. Numerical analysis of multilayer model also includes the boundary conditions to show multilayer model. Differentiating of the voltage also demonstrates how to obtain the electric field intensity for multilayer soil profile. This paper first derives the theoretical equations for calculating earth resistivity in order to prepare the resistivity curves for layer-by-layer analysis. It also describes the features of resistivity curves in a multilayer structure and some numerical analysis for estimating earth parameters. Finally, the model for estimating earth parameters is presented through potential differences and resistivity ratio in a multilayer earth structure.

The model based on soil resistivity ratio and voltage difference of multilayered earth structure is able to define soil characteristics with more specifications in geotechnical profile. Including potential differences and thickness in the model shows the reliable and efficient outcomes in near surface soil investigations. The aim of the research is demonstration of the model for obtaining more specific soil characterizations through multilayer soil resistivity profile.

2. Wenner Apparent Resistivity

The basic equation for the electric potentiality and soil resistivity [11] is also included in the study. The equation of the electric filed is given corresponding to the gradient of scalar potential as,

$$E = -\text{grad}\,\psi. \tag{2.1}$$

Another basic equation of current density, **J** is that

$$\text{div}\,\mathbf{J} = 0. \tag{2.2}$$

The conductivity, $\sigma(z)$, varies according to the depth, z. Hence, the partial differential equation of electrical potentiality is expressed as,

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{1}{\sigma}\left[\frac{\partial\psi}{\partial x}\frac{\partial\sigma}{\partial x} + \frac{\partial\psi}{\partial y}\frac{\partial\sigma}{\partial y} + \frac{\partial\psi}{\partial z}\frac{\partial\sigma}{\partial z}\right] = 0. \tag{2.3}$$

According to cylindrical coordinates (r, z), we obtain

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{\sigma}\frac{\partial\psi}{\partial r}\frac{\partial\sigma}{\partial r} + \frac{\partial^2\psi}{\partial z^2} + \frac{\partial\psi}{\partial z}\frac{1}{\sigma}\frac{\partial\sigma}{\partial z} = 0, \tag{2.4}$$

where $r = \sqrt{x^2 + y^2}$. Here, $(\partial\psi/\partial r)(\partial\sigma/\partial r)$ in (2.4) vanishes as consideration of conductivity, $\sigma(z)$, varies according to depth z.

Due to the azimuthal symmetry, we can separate (2.4) using separation of variables as $\psi(r, z) = \widehat{\mathbf{R}}(r)Z(z)$, which is a product of a function of r and a function of z. Thus,

$$\frac{d^2\widehat{\mathbf{R}}}{dr^2} + \frac{1}{r}\frac{d\widehat{\mathbf{R}}}{dr} + \lambda^2\widehat{\mathbf{R}} = 0, \tag{2.5}$$

where $\widehat{\mathbf{R}}$ is the vector of cylindrical coordinates.

And

$$\frac{d^2 Z}{dz^2} + \frac{1}{\sigma}\frac{d\sigma}{dz}\frac{dZ}{dz} - \lambda^2 Z = 0, \tag{2.6}$$

where λ is the separation constant.

Thus, a general solution on electric potential can be written as (2.7)

$$\psi = \int_0^\infty F(\lambda)\widehat{\mathbf{R}}(\lambda, r)Z(\lambda, z)d\lambda. \tag{2.7}$$

In addition, the electric potential and electric intensity being varied according to depth is considered as a function of z.

Now, the potentiality equation can be derived from (2.7) with Bessel function theory for layer 1 and 2 as,

$$\psi_1 = \frac{I\rho_1}{2\pi}\int_0^\infty \left[f_1(\lambda)e^{-\lambda z} + g_1(\lambda)e^{\lambda z}\right]J_0(\lambda r)d\lambda, \tag{2.8}$$

$$\psi_2 = \frac{I\rho_1}{2\pi}\int_0^\infty \left[f_2(\lambda)e^{-\lambda z} + g_2(\lambda)e^{\lambda z}\right]J_0(\lambda r)d\lambda, \tag{2.9}$$

where, ρ is the soil resistivity. Thus, $f_1(\lambda)$ and $g_1(\lambda)$, $f_2(\lambda)$ and $g_2(\lambda)$ can be solved with following boundary conditions:

at $z \to \infty$ $\psi_2 \to 0$,

at $z \to 0$ $\partial\psi_1/\partial z = 0$,

at $z = h$ $\psi_1 = \psi_2$,

at $z = h (1/\rho_1)(\partial\psi_1/\partial z) = (1/\rho_2)(\partial\psi_2/\partial z)$.

Now, $f_1(\lambda)$ and $g_1(\lambda)$ are obtained according to the boundary condition:

$$f_1(\lambda) = \frac{Ke^{-2\lambda h}}{1 - Ke^{-2\lambda h}}, \qquad g_1(\lambda) = f_1(\lambda), \tag{2.10}$$

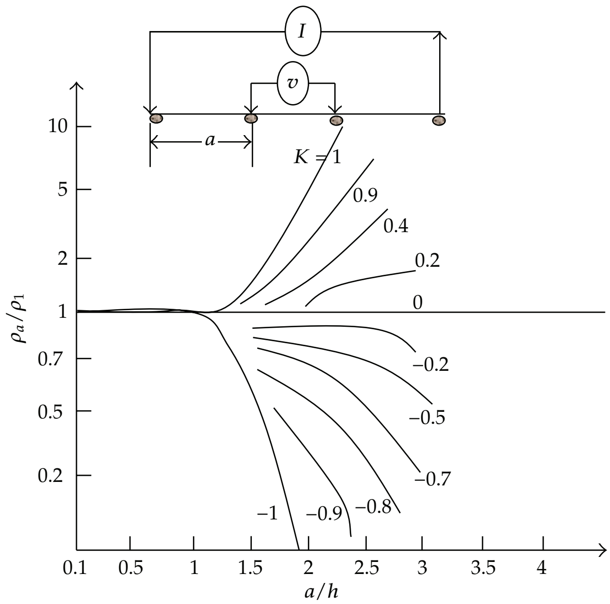

Figure 2: Resistivity ratio for two-layer model using Wenner method.

where reflection coefficient, K, is defined as

$$K = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}. \tag{2.11}$$

In the Wenner array, four electrodes are arranged in a straight line with equal spacing of a [12]. For the two-layer model, the ratio of apparent resistivity, ρ_a, and upper layer resistivity are estimated according to the relative spacing a/h including various values of the factor, K, as shown in Figure 2. Curves of previously used two-layer model include upper layer resistivity, ρ_1, and apparent resistivity, ρ_a, of having electrical properties in near surface soil investigations.

3. Methodology

The research study on the soil characterizations with multilayer electric model is conducted by geoelectric research group at Geotechnical Laboratory, Faculty of Engineering and Built Environment, University Kebangsaan Malaysia (UKM). The analysis for multilayer soil characterizations also includes BEM numerical analysis. Simulations for multilayer layer earth structures are conducted with partial differential equation (PDE) tools in Matlab 2009 including boundary element formulation. Simulations with PDE tools are done for the conditions of conductive media DC for electric field generations in soil. The data collection for obtaining multilayer electrical properties with four-probe Wenner method is carried out.

In the study, the parameters of soil resistivity, potential differences, and thickness of each layer are considered for the implementation of multilayer earth model as Figure 3. Four-probe resistivity method is used to estimate soil electric properties in geotechnical investigations [13]. When current is injected in the soil, electric field is generated in the near

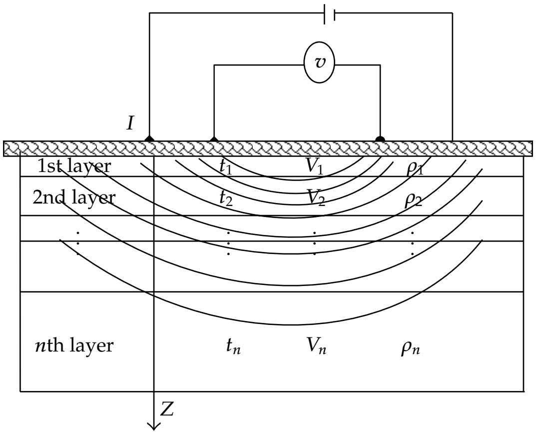

Figure 3: Multilayer earth structure in soil characterizations.

surface soil. The potentiality and electric field also vary according to changing of depth in soil [14].

The n potential functions can be written as n layers model for normal surface in multilayer earth structure. The derivation of potential function of the uppermost layer can be revealed with Bessel function:

$$\psi_0 = \frac{\rho_0 I}{2\pi}\left[\frac{1}{(r^2+z^2)^{1/2}} + \int_0^{\infty} B_0(m) J_0(mr) e^{mz} dm\right]. \tag{3.1}$$

The potentiality of ith layer model is shown with (3.2):

$$\psi_i = \frac{\rho_i I}{2\pi}\left[\int_0^{\infty} A_i(m) J_0(mr) e^{-mz} dm + \int_0^{\infty} B_i(m) J_0(mr) e^{mz} dm\right]. \tag{3.2}$$

Moreover, the potentiality of nth layer is written as

$$\psi_n = \frac{\rho_n I}{2\pi}\left[\int_0^{\infty} A_n(m) J_0(mr) e^{-mz} dm\right]. \tag{3.3}$$

A set of n equations is obtained when that potential is continuous across each boundary. The potential equation for multilayer soil profile also shows the derivation of resistivity with Bessel function as,

$$\rho_0\left[\int_0^{\infty} J_0(mr) e^{-mz_1} dm + \int_0^{\infty} B_0(m) J_0(mr) e^{mz_1} dm\right]$$
$$= \rho_1\left[\int_0^{\infty} A_1 J_0(mr) e^{-mz_1} dm + \int_0^{\infty} B_1(m) J_0(mr) e^{mz_1} dm\right],$$

$$\rho_n\left[\int_0^\infty A_n(m)J_0(mr)e^{-mz_n}dm+\int_0^\infty B_n(m)J_0(mr)e^{mz_n}dm\right]$$
$$=\rho_{n+1}\left[\int_0^\infty A_{n+1}(m)J_0(mr)e^{-mz_n}dm\right]. \tag{3.4}$$

In addition, a second set of n equations is formed by equating the components of current density normal to the boundary for each boundary:

$$-\int_0^\infty mJ_0(mr)e^{-mz_1}dm+\int_0^\infty mB_0(m)J_0(mr)e^{mz_1}dm$$
$$=-\int_0^\infty mA_1(m)J_0(mr)e^{-mz_1}dm+\int_0^\infty mB_1(m)J_0(mr)e^{mz_1}dm,$$
$$-\int_0^\infty mA_n(m)J_0(mr)e^{-mz_n}dm+\int_0^\infty mB_n(m)J_0(mr)e^{mz_n}dm=-\int_0^\infty mA_{n+1}(m)J_0(mr)e^{-mz_n}dm. \tag{3.5}$$

All the integrations are carried out through the same limits. Thus, (3.4) can be written as

$$\int_0^\infty\left\{\left[e^{-mz_1}+B_0(m)e^{mz_1}\right]\rho_0-\left[A_1(m)e^{-mz_1}+B_1(m)e^{mz_1}\right]\rho_1\right\}J_0(mr)dm=0,$$
$$\int_0^\infty\left\{\left[A_n(m)e^{-mz_n}+B_n(m)e^{mz_n}\right]\rho_n-\left[A_{n+1}(m)e^{-mz_n}\right]\rho_{n+1}\right\}J_0(mr)dm=0. \tag{3.6}$$

And, we get from (3.5) as

$$\int_0^\infty\left[-e^{-mz_1}+B_0(m)e^{mz_1}+A_1(m)e^{-mz_1}-B_1(m)e^{mz_1}\right]J_0(mr)mdm=0,$$
$$\int_0^\infty\left[-A_n(m)e^{-mz_n}+B_n(m)e^{mz_n}+A_{n+1}(m)e^{-mz_n}\right]J_0(mr)mdm=0. \tag{3.7}$$

These equations hold for all points along various planes separating the layers

$$\rho_0e^{-mz_1}+\rho_0B_0(m)e^{mz_1}-\rho_1A_1(m)e^{-mz_1}-\rho_1B_1(m)e^{mz_1}=0,$$
$$-e^{-mz_1}+B_0(m)e^{mz_1}+A_1(m)e^{-mz_1}-B_1(m)e^{mz_1}=0,$$
$$\rho_nA_ne^{-mz_n}+\rho_nB_n(m)e^{mz_n}-\rho_{n+1}A_{n+1}(m)e^{-mz_n}=0,$$
$$-A_ne^{-mz_n}+B_n(m)e^{mz_n}+A_{n+1}(m)e^{-mz_n}=0. \tag{3.8}$$

These sets of equations are solved directly for all the A and B parameters by working from the above equations:

$$A_{n+1} = A_n - Be^{2mz_n},$$
$$B_n = \frac{\rho_{n+1}}{\rho_n}\left(A_n - Be^{2mz_n}\right)e^{-2mz_n} - A_n e^{-2mz_n}. \tag{3.9}$$

The kernel function of the integration, $A_1(m) + B_1(m)$ for the multilayer soil structure, is obtained which is related to the resistivity depth function $\rho(z)$. The multilayer reflection coefficient is shown as

$$K_1 = \frac{\rho_{n+1} - \rho_n}{\rho_{n+1} + \rho_n}. \tag{3.10}$$

With subtraction and summation of the resistivity, the multilayer reflection coefficient can be represented as

$$K_1' = \frac{\rho_{n+1}}{\rho_n}. \tag{3.11}$$

Here, multilayer reflection coefficient, K_1', is derived to get the function of resistivity ration for the adjacent two layers in multilayer resistivity model.

The algorithm involving iterative search for multilayer layer earth structure is given below

(1) Estimate soil resistance, R, resistivity, ρ, voltage, v, and thickness, t with four-probe soil resistivity measurements technique.

(2) Obtain n layers according to resistivity and potential data in soil electric field.

(3) Apply boundary element formulations with k nodes for each layer and iteration are implemented using array technique.

(4) For the current values of soil parameters, ρ_n^k, v_n^k, t_n^k, the elements of the $n \times k$ matrices are computed:

$$V_{ij} = \begin{vmatrix} \frac{dv_{11}}{dh_{11}} & \frac{dv_{12}}{dh_{12}} & \cdots & \frac{dv_{1k}}{dh_{1k}} \\ \frac{dv_{21}}{dh_{21}} & \frac{dv_{22}}{dh_{22}} & \cdots & \frac{dv_{2k}}{dh_{1k}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{dv_{n1}}{dh_{n1}} & \frac{dv_{n2}}{dh_{n2}} & \cdots & \frac{dv_{nk}}{dh_{nk}} \end{vmatrix},$$
$$\Re_{ij} = \begin{vmatrix} \frac{\rho_{2,1}}{\rho_{1,1}} & \frac{\rho_{2,2}}{\rho_{1,2}} & \cdots & \frac{\rho_{2,k}}{\rho_{1,k}} \\ \frac{\rho_{3,1}}{\rho_{2,1}} & \frac{\rho_{3,2}}{\rho_{2,2}} & \cdots & \frac{\rho_{3,k}}{\rho_{2,k}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\rho_{n+1,1}}{\rho_{n,1}} & \frac{\rho_{n+1,2}}{\rho_{n,2}} & \cdots & \frac{\rho_{n+1,k}}{\rho_{n,k}} \end{vmatrix}. \tag{3.12}$$

(5) The updated soil parameters are put in the array with the element of T as

$$x^T = [V, \Re, t]^T. \tag{3.13}$$

(6) Step 4 and 5 will be repeated until the ending of k numbers of nodes and n numbers of layer in soil multilayer profile. Threshold, $\mathfrak{I}$ is set to take adjustment in the iteration of multilayer model.

BEM is applied to obtain multilayer model with boundary conditions in soil investigations. BEM criteria [15, 16] is applied to the electric potential of the soil as the solution of Poisson equation is shown in (3.14)

$$-\nabla \cdot \left(\frac{1}{\rho} \nabla \varphi_i\right) = Q, \quad (i = 0, 1, 2, 3), \tag{3.14}$$

where φ_i is the electric potential in volts at coordinates (x, y), ρ is resistivity and Q is the current source. BEM is also used in the proposed multilayer model to derive electric potential in terms of potential functions φ_i based on cylindrical coordinate system.

4. Results and Discussions

Recent advancements in electronics and electrical engineering have improved the ability to obtain near surface soil characteristics with electrical properties in geotechnical engineering. Four-probe Wenner method is commonly used with two-layer earth model for soil characterizations through electric field generation in soil.

Multilayer earth structure reveals the profile of more specific soil properties with different soil electric parameters and has made possible the investigation of techniques for estimating these parameters. Soil resistivity and soil potentiality are obtained through the creation of soil electric field in near surface soil. The study of the soil electric properties for different layer of soil is needed to obtain reliable outcomes in soil characterization.

Figure 4 shows the simulation results for multilayer soil electric field in near surface soil profile. In the present work, we focus our attention on multilayer soil electric model of the electric potentiality and soil resistivity in grounding system. The numerical approach based on the BEM for layered soil model is presented here. Potential measurements on the near surface soil between the metallic probes are taken when the current is injected into the soil.

In the numerical analysis, the soil is modeled in multilayer according to the soil characterization method. The bottom layer is considered as a half-space domain and bounded with the implementation of boundary condition. The parameters of thickness of soil layers are considered for multilayer model including the interface of soil layer and probes.

Soil resistivity is obtained with the consideration of voltage differences between two probes and injected current using four-probe electric method. The potentiality and electric field strength also decrease according to the increasing of the depth of soil profile shown in (2.8). The soil profile is modeled in multilayer for numerical analysis according to the collected soil resistivity in soil electric field investigations. Figure 5 shows the variation of resistivity with depth in a multilayer soil profile.

The theoretical derivations for multilayer soil resistivity profile are shown based on the wave propagation solution of electrical engineering. Reflection coefficient K_1' is derived

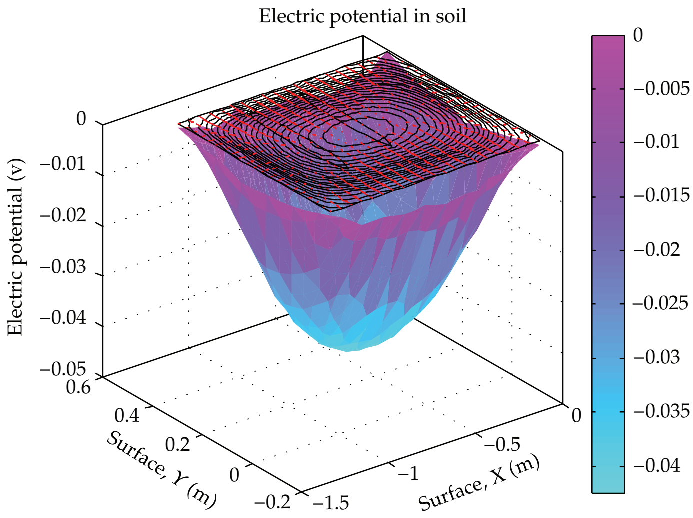

Figure 4: 3D profile of soil electric potential in near surface soil.

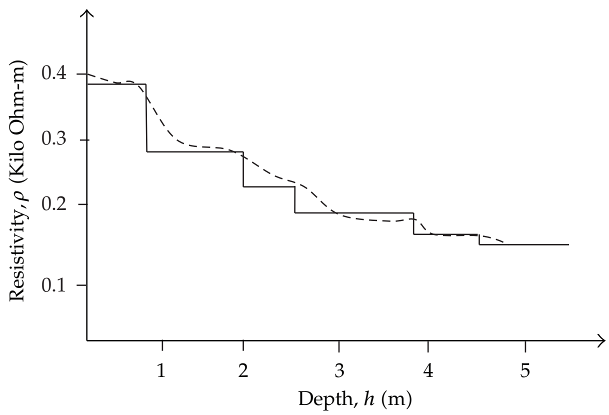

Figure 5: Multiple-layer considerations according to collected soil resistivity data.

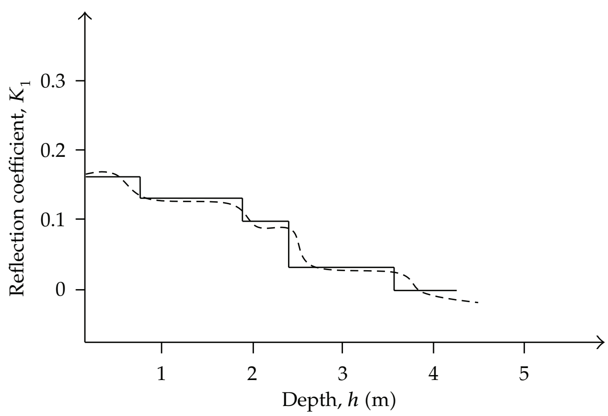

Figure 6: Plot of multilayer reflection coefficient with depth.

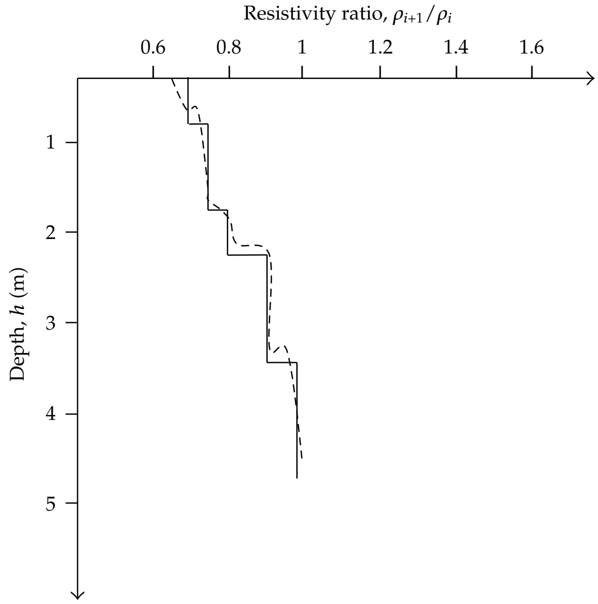

Figure 7: Depth corresponding resistivity ratio in soil investigations.

in methodology of the study as the resistivity ratio of two adjacent layers in multilayer soil electric model. The resistivity ratio and potential differences for the adjacent two layers are considered in the new proposed model of multilayer soil structure. The variation of reflection coefficient with depth is shown in Figure 6.

In addition, the resistivity ratio is considered for the implementation of proposed multilayer soil electric model. The results of numerical analysis including resistivity ratio are revealed through Figure 7 for multilayer earth structure on soil characterizations. In the half space of the multilayer soil electric model, the thickness is considered as infinity for near surface soil profile. The resistivity ratio of the half space is not considerable to get a simple calculation in multilayer soil investigations. As an example, the resistivity ratio, ρ_2/ρ_1 is demonstrated as 0.73, where resistivity, ρ_1 considered as 0.38 Kilo Ohm-m and ρ_2 is as 0.28 Kilo Ohm-m for easy analyzing in the multi-layer earth structure. The resistivity of third layerρ_3 is taken as 0.22 Kilo Ohm-m that manifests the resistivity ratio, ρ_3/ρ_2 as 0.78.

However, there are the effects of soil type, water contents, bulk density of soil on resistivity profile of soil. Consideration of only resistivity profile for soil characterizations can vary for the presence of water contents. Thus, the resistivity ratio for multilayer is considered in proposed model to obtain reliable and specific near surface soil characteristics with electrical properties.

Seedher and Arora [16] shows two layer model for measurements of near surface soil resistivity. This two-layer model using four-probe Wenner method also considers the apparent resistivity which can show unreliable results of near surface soil profile. Moreover, there are no specific characterizations for different layers of near surface soil with two-layer soil resistivity model.

There is the implementation of BEM by De Lacerda et al. [17] for the analysis of cathodic protection systems of buried slender structures. In that paper, the soil resistivity properties are measured along the depth where two-layer apparent resistivity model has been used. In addition, Binley and Kemna [18] shows the implementation of DC resistivity for soil characterizations. They demonstrated the surface imaging with soil apparent resistivity using Wenner method. Though apparent resistivity can be used to obtain deeper soil profile, there are limitations in obtaining accurate results for empirical relationship between probe distances and depth of corresponding soil profile. In addition, considering soil apparent resistivity can affect the results of near surface soil investigations.

Therefore, there is concentration on multilayer true resistivity model to obtain accurate results in soil investigations. The theoretical derivations and numerical analysis of multilayer true resistivity model are demonstrated for the application of near surface soil characterizations. Moreover, as the experience of using this model, it is easy to set up in field as there is no need for excavation of hole in surface soil in geotechnical investigations which is tedious and time consuming.

5. Conclusion

Multilayer soil electric model is demonstrated for soil electrical characterizations with robust and more reliable performance in near surface soil profile. The theoretical equations with numerical analysis on soil electric field are derived for multilayer soil investigations. The feasibility of implementation of resistivity ratio and potential difference for robust multilayer soil profile is shown with numerical analysis of electrical properties. The boundary element formulations are considered in the numerical analysis on layered soil models. The electrical properties with soil multilayer profile are crucial to obtain specific and reliable outcomes for getting depth corresponding soil characteristics in geoelectric engineering. This multilayer soil electric model would be the improvement in obtaining bulk density of compacted soil and detecting hazardous organic waste in geotechnical engineering.

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References

[1] S. P. Friedman, "Soil properties influencing apparent electrical conductivity: a review," *Computers and Electronics in Agriculture*, vol. 46, no. 1–3, pp. 45–70, 2005.
[2] A. Osella and A. Favetto, "Effects of soil resistivity on currents induced on pipelines," *Journal of Applied Geophysics*, vol. 44, no. 4, pp. 303–312, 2000.
[3] T. Saarenketo, "Electrical properties of water in clay and silty soils," *Journal of Applied Geophysics*, vol. 40, no. 1–3, pp. 73–88, 1998.
[4] Y. Son, M. Oh, and S. Lee, "Estimation of soil weathering degree using electrical resistivity," *Environmental Earth Sciences*, vol. 59, no. 6, pp. 1319–1326, 2010.
[5] J. M. Sabatier, D. C. Sokol, C. K. Frederickson, M. J. M. Romkens, E. H. Grissinger, and J. C. Shipps, "Probe microphone instrumentation for determining soil physical properties: testing in model porous materials," *Soil Technology*, vol. 8, no. 4, pp. 259–274, 1996.
[6] G. L. Yoon and J. B. Park, "Sensitivity of leachate and fine contents on electrical resistivity variations of sandy soils," *Journal of Hazardous Materials*, vol. 84, no. 2-3, pp. 147–161, 2001.
[7] J. A. Doolittle, K. A. Sudduth, N. R. Kitchen, and S. J. Indorante, "Estimating depths to claypans using electromagnetic induction methods," *Journal of Soil & Water Conservation*, vol. 49, no. 6, pp. 572–575, 1994.

[8] J. D. McNeill, "Electromagnetic terrain conductivity measurement at low induction numbers," Technical note TN-6, Geonics Limited, Ontario, Canada, 1980, http://www.geonics.com/pdfs/technicalnotes/tn6.pdf.
[9] M. H. Zhao, P. Z. Fang, and Y. P. Shen, "Boundary integral-differential equations and boundary element method for interfacial cracks in three-dimensional piezoelectric media," *Engineering Analysis with Boundary Elements*, vol. 28, no. 7, pp. 753–762, 2004.
[10] I. Colominas, F. Navarrina, and M. Casteleiro, "Boundary element formulation for the analysis of transferred potentials in electrical installations," *Advances in Engineering Software*, vol. 35, no. 10-11, pp. 601–607, 2004.
[11] X. Lu, D. L. Alumbaugh, and C. J. Weiss, "The electric fields and currents produced by induction logging instruments in anisotropic media," *Geophysics*, vol. 67, no. 2, pp. 478–483, 2002.
[12] M. Séger, I. Cousin, A. Frison, H. Boizard, and G. Richard, "Characterisation of the structural heterogeneity of the soil tilled layer by using in situ 2D and 3D electrical resistivity measurements," *Soil and Tillage Research*, vol. 103, no. 2, pp. 387–398, 2009.
[13] A. Samouëlian, I. Cousin, A. Tabbagh, A. Bruand, and G. Richard, "Electrical resistivity survey in soil science: a review," *Soil and Tillage Research*, vol. 83, no. 2, pp. 173–193, 2005.
[14] R. K. Frohlich, P. J. Barosh, and T. Boving, "Investigating changes of electrical characteristics of the saturated zone affected by hazardous organic waste," *Journal of Applied Geophysics*, vol. 64, no. 1-2, pp. 25–36, 2008.
[15] S. Yildirim, "Exact and numerical solutions of poisson equation for electrostatic potential problems," *Mathematical Problems in Engineering*, vol. 2008, Article ID 578723, 2008.
[16] H. R. Seedher and J. K. Arora, "Estimation of two layer soil parameters using finite Wenner resistivity expressions," *IEEE Transactions on Power Delivery*, vol. 7, no. 3, pp. 1213–1217, 1992.
[17] L. A. De Lacerda, J. M. Da Silva, and J. Lázaris, "Dual boundary element formulation for half-space cathodic protection analysis," *Engineering Analysis with Boundary Elements*, vol. 31, no. 6, pp. 559–567, 2007.
[18] A. Binley and A. Kemna, "DC resistivity and induced polarization methods," *Hydrogeophysics*, vol. 50, no. 2, pp. 129–156, 2005.


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